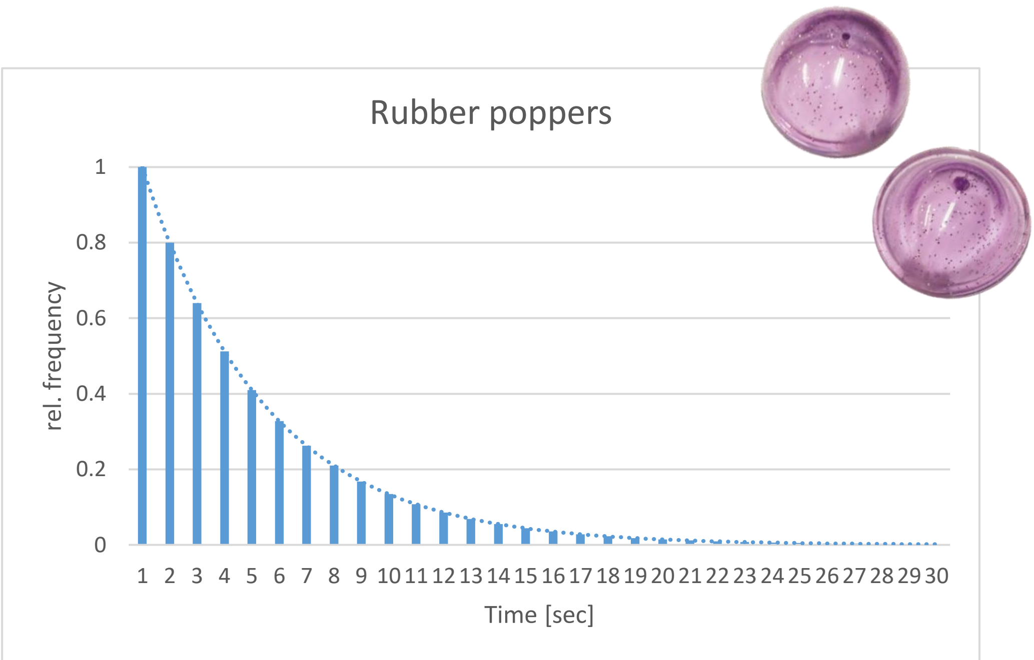
Introduction of radioactivity by using probability theory

Annotated learning unit

This learning unit was translated from the original German to English and summarized accordingly.



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I) Probability theory

In probability theory, random phenomena are modeled through mathematical models. We can make predictions about random events with the help of probability theory, for example with the law of large numbers.

The law of large numbers…

…says, if a random experiment is conducted very often in succession, or many similar experiments are conducted at the same time, the relative frequency of an output corresponds to the probability of that output.

Key idea 1

A single random event is unpredictable, but many random events follow a pattern.

For example, rolling a die is a **random experiment**. Because the outcome on which number the die will land cannot be predicted, one can only make probability statements.

The example of rolling a die to illustrate the law of large numbers.

By rolling a die very often and noting how often which number appears, one can find a **distribution**. While for a single roll the outcome is unpredictable, such a distribution (with enough repetitions) always looks the same.

The total number of times of one of the different outcomes (i.e. when rolling a die, how often a one, a two, etc., appears) is called **absolute frequency**. The absolute frequency divided by the number of repetitions is called **relative frequency.** According to the law of large numbers, the relative frequency corresponds to the probability of an outcome. The distribution assigning every outcome of a random experiment to a relative frequency is called a **probability distribution.**

The relative frequency of an random event corresponds to the probability.

Two examples:

1. Roll the die

When rolling a die very often, all outcomes will appear about the same number of times. Therefore the probability for each outcome is the same. The outcomes are **equally distributed.**

Examples for an equally and a non equally distributed random experiment.

2. Thumbtacks

When throwing thumbtacks, the distribution would be different. It is more likely that the thumbtack lands on the pointy side than on the flat side. The outcomes are **not equally distributed.**

The “time problem”

Key idea 2

Depending on the definition oft he random event, the probability distribution is different.

When rolling a die or throwing a thumbtack very often and count how often one outcome appears, one determines, so to speak, a **probability per throw/roll**.

Difference between probability per time and probability per roll.

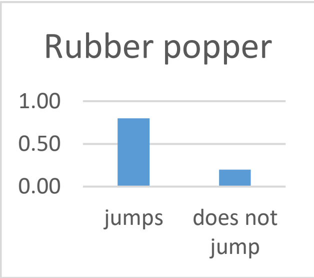
In the case of such discrete repetitions, this is still relatively easy. However, it becomes more difficult to find a **probability per time,** for example, with rubber poppers.

Rubber poppers are a hollow hemisphere made of an elastic material (example on the left of the picture). They can be folded (example on the right of the picture) and placed on the table; after a few seconds, the tension will be released, and the popper will jump.

The rubber popper is an example to determine the probability per time.

For them, it is random when (in which time interval) they jump. Therefore rubber poppers have a certain jump probability that is the same for every time interval.

For example, if the rubber popper jumps with a probability of 2/10 per time interval, the probability distribution for each time interval would look as shown.



To determine the jump probability in a specific time interval, one needs to count the number of jumped rubber poppers in a particular interval. Again, after many repetitions, you will be able to determine a jump probability distribution per time, i.e. the probability with which it jumps in a time interval.

How to determine the probability per time.

Since time passes continuously, it is usually divided into intervals, for example seconds. In the case of the rubber popper, 5-second intervals were used.

Justification for using time intervalls in the case of a rubber popper.

II) The radioactive transformation

So far we have used different models to make probability theory a little more understandable with the law of large numbers. The macroscopic model with the rubber poppers is, however, well suited to model the radioactive decay of an unstable atomic nucleus.

Stable Nuclei and Unstable Nuclei

The radioactive decay of an unstable atomic nucleus can be modeled with the rubber popper: A folded rubber popper is unstable. Once it has jumped, it has transformed, i.e., it is stable. There are atomic nuclei that are unstable due to their internal structure, i.e., they spontaneously decay. In this context, decaying means "transforming into a more stable state". That is, if the unstable atomic nucleus is stable after a particle has been emitted, it will not decay any further.

Rubber popper can be used to illustrate stable and unstable nuclei.

Key idea 1

The transformation of an unstable nucleus is not a process, but happens spontaneously.

It would be better to use "transformation" instead of "decay", because the atomic nucleus neither falls apart into individual pieces nor continuously decays until nothing is left, but it transforms by emitting a particle.

Use „transformation“ instead of ~~„decay“~~.

In this unit, only the simplest case is considered: After one transformation, the unstable nucleus is stable, like it is the case with the rubber poppers.

Start with only one tranformation until stable.

The jump probability per time is now called the **decay probability**, i.e., a probability that an unstable nucleus will transform in a time interval. This probability is always the same for atoms of the same element – it is a natural constant.

Key idea 2

How many unstable nuclei of an element transform can be predicted. The time when a particular nucleus trandforms, however, is random.

However, different elements have a different decay probability. The point in time when an individual atomic nucleus transforms is random. With many unstable nuclei of the same element, one can determine the decay probability using the law of large numbers.

The decay probability is a probability per time and can be determined with the law of large numbers

III) Half life time

Key idea 1

For a source with many unstable nuclei, the half-life indicates the time in which half of the unstable nuclei transformed.

The half-life is the time after which half of all unstable nuclei in an ensemble have transformed. One can also calculate the half-life of thumbtacks.

Sorting out thumbtacks

Consider throwing thumbtacks as a model for the radioactive transformation. Thumbtacks landing on the flat side have transformed and cannot transform again. In a jar are a vast amount of thumbtacks. They are thrown all at once. The unstable ones (those who landed on the pointy side) are counted, returned to the jar, and thrown again. This experiment is repeated until no thumbtacks are left in the jar. After each throw, the number of thumbtacks in the jar will be reduced by 1/3. At the beginning, all thumbtacks are “unstable”, and there is a 1/3 probability that they will land on the flat side with the first throw. At the next throw, 1/3 of the thumbtacks are already "stable" and are no longer thrown, etc.

In this case, the half-life would not be a time at all but would lie between the first and second throw. It is rather a half-life throw or repetition.

Thumbtacks are an example of a random experiment, where one can determine a half-life time that is no time but a repitition or throw.

Half life of rubber poppers

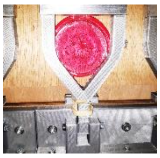
The model of the rubber poppers can also be used to illustrate unstable atomic nuclei by counting how many remain per second, i.e. are still "unstable".

Performing the same experiment as with the thumbtacks above with rubber poppers is difficult. Because one would need to fold and release a vast amount of rubber poppers at the same time. In this learning unit, a device was developed, that allows to release 10 rubber poppers at the same time. Students can then already guess what the distribution looks like for a bigger amount of rubber poppers.

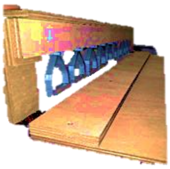
Rubber poppers are an example of a random experiment, where you can determin a half life time.

The rubber popper device

With the rubber popper device, it is possible to release ten rubber poppers at the same time. The device consists of ten 3D printed plastic traps (circled in green) and an upper and lower wooden structure. These wooden structures are connected with hinges (blue arrows) and can be opened by removing a screw in the front (circled in yellow).



In each trap, one folded rubber popper can be clamped between the wooden and the plastic parts, like in the picture on the left.



When all ten rubber poppers are in position, one can open the maschine by removing the screw, like in the picture on the right.